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Energy-Momentum Distributions of Hawking Wormholes

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Abstract This study is purposed to elaborate the problem of energy and momentum distributions of the Hawking wormholes in general theory of relativity and teleparallel gravity. In this connection, the energy and momentum for the Hawking wormhole space-time are evaluated using Einstein, Bergmann-Thomson, Landau-Lifshitz and Möller energy-momentum prescriptions in these approximations. We obtained that some of these energy-momentum distributions of Hawking wormhole in the form of general relativity (GR) and teleparallel gravity (TG) give the same results. Also, we obtained that in the teleparallel gravity, Möller energy distributions of the Hawking wormholes give the same results as Einstein energy distributions. However, Bergmann-Thomson and Landau-Lifshitz energy-momentum of Hawking wormholes are the different from each others.

Keywords Hawking wormhole · Energy-momentum complexes · General relativity · Teleparallel gravity

1 Introduction

It is well known that one of the most interesting and challenging problems of general relativity is the energy and momentum localization. Energy-momentum is an important conserved quantity in any physical theory whose definition has been under investigation for a long time from the General Relativity viewpoint. The problem is to find an expression which is physically meaningful. The point is that the gravitational field can be made locally vanish and so one is always able to find the frame in which the energy-momentum of gravitational field is zero while in the other frames, it is not true. Unfortunately, there is still no generally

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accepted definition of energy-momentum for gravitational field. The problem arises with the expression defining the gravitational field energy part.

In the theory of General Relativity, the energy-momentum conservation laws are given by

$$T^b_{a;b} = 0 \quad (a, b = 0, 1, 2, 3), \tag{1}$$

where T^b_a denotes the energy-momentum tensor. In order to change the covariant divergence into an ordinary divergence so that global energy-momentum conservation, including the contribution from gravity, can be expressed in the usual manner as in electromagnetism, Einstein formulated [1] the conservation law in the following form

$$\frac{\partial}{\partial x^b}(\sqrt{-g}(T^b_a + t^b_a)) = 0. \tag{2}$$

Here t^b_a is not a tensor quantity and is called the gravitational field pseudo-tensor. Schrödinger showed that the pseudo-tensor can be made vanish outside the Schwarzschild radius using a suitable choice of coordinates. There have been many attempts in order to find a more suitable quantity for describing the distribution of energy and momentum due to matter, non-gravitational and gravitational fields. The proposed quantities which actually fulfill the conservation law of matter plus gravitational parts are called gravitational field complexes. The choice of the gravitational field complexes is not unique. Because of this, quite a few definitions of these complexes have been proposed. The notion of energy-momentum prescriptions was severely criticized for a number reasons. Firstly, the nature of symmetric and locally conserved object is non-tensorial one; thus its physical interpretation appeared obscure [2]. Secondly, different energy-momentum complexes could yield different energy-momentum distributions for the same gravitational background [3]. Finally, energy-momentum complexes were local objects while it was generally believed that the suitable energy-momentum of the gravitational field was only total, i.e. it cannot be localized [4]. There have been several attempts to calculate energy-momentum prescriptions associated with different space-times [5, 6]. Virbhadra [7] showed that the definitions of Einstein, Tolman and Landau and Lifshitz give the same energy distribution for the Kerr-Newman metric. Later, Aguirregabiria et al. [8] proved that definitions of Einstein, Landau and Lifshitz, Weinberg and Papapetrou give the same result for any metric of Kerr-Schild class. Later, Virbhadra [9] emphasized that these complexes in fact coincide for space-times more general than the Kerr-Schild class. He also computed energy distribution for a general non-static spherically symmetric space-time of Kerr-Schild class and found that all these definitions give the same result as given by the Penrose quasi-local definition of energy.

In order to obtain a meaningful expression for energy, momentum and angular momentum for a general relativistic system, Einstein himself proposed an expression. After Einstein's energy-momentum complex [10], many complexes have been found, for instance, Landau-Lifshitz [11], Tolman [12], Papapetrou [13], Möller [14, 15], Weinberg [16] and Bergmann-Thomson [17]. Some of these definitions are coordinate dependent while others are not. There lies a dispute on the importance of non-tensorial energy-momentum complexes whose physical interpretations have been questioned by a number of physicists, including Weyl, Pauli and Eddington. Also, there exists an opinion that the energy-momentum pseudo-tensors are not useful to find meaningful results in a given geometry. Ever since the Einstein's energy-momentum complex [18], used for calculating energy and momentum in a general relativistic system, many attempts have been made to evaluate the energy distribution for a given space-time [12]. Except for the one which was defined by Möller, these definitions only give meaningful results if the calculations are performed in "quasi-Cartesian" coordinates. Möller constructed an expression which enables one to evaluate energy and momentum in any coordinate system. Lessner [19] argued that the Möller prescription is a powerful concept of energy-momentum in general relativity.

Recently, some authors have studied Weinberg energy-momentum complexes for a stringy black hole [20] and Energy distribution of Schwarzschild-like space-time [21]. Vagenas [22] has studied energy distribution of Reissner-Nordström Blackhole in Möller prescription. Some authors have studied on the Möller energy associated with black-holes [23] and Energy of charged wormholes [24] and also, Möller's energy of charged black-hole [25]. Furthermore, Möller energy distributions of various wormholes have been studied by Aygun and Yilmaz [26] and energy-momentum of rigidly rotating wormhole space-time in GR have been studied by Aygun et al. [27].

The wormhole solutions of the Einstein equations started with Einstein himself, since he was interested in giving a field representation of particles [28]. The idea was further developed by Ellis [29] and others, where instead of particles, they try to model them as "bridges" between two regions of the space-time. The idea of considering such solutions a actual connections between two separated regions of the Universe has attracted a lot of attention since the seminal work of Morris and Thorne [30]. For the Lorentzian wormhole to be traversable, it requires exotic matter which violates the known energy conditions. In this paper, we calculate the Einstein, Bergmann-Thomson, Landau-Lifshitz and Möller energy-momentum distributions of the Hawking wormhole space-time in general relativity and teleparallel gravity.

We will proceed according to the following scheme. In Sect. 2, we give simple definitions of Einstein, Bergmann-Thomson, Landau-Lifshitz and Möller prescriptions in general relativity and teleparallel gravity. In Sect. 3, we get energy-momentum distributions of Hawking wormhole in general relativity and teleparallel gravity. Finally, we summarize and discuss our results.

2 Some Energy-Momentum Prescriptions in the General Relativity

2.1 Bergmann-Thomson Energy-Momentum Prescription in GR

The energy-momentum prescription of Bergmann-Thomson [17] is given by

$$B^{\mu\nu}_{(\mathrm{GR})} = \frac{1}{16\pi} \Pi^{\mu\nu\beta}_{,\beta} \tag{3}$$

where

$$\Pi^{\mu\nu\beta} = g^{\mu\alpha} V^{\nu\beta}_{\alpha} \tag{4}$$

with

$$V^{\nu\alpha}_{\beta} = -V^{\alpha\nu}_{\beta} = \frac{g_{\beta\xi}}{\sqrt{-g}} [-g(g^{\nu\xi} g^{\alpha\rho} - g^{\alpha\xi} g^{\nu\rho})]_{,\rho}, \tag{5}$$

$B^{00}_{(\mathrm{GR})}$ is the energy density, $B^{\mu 0}_{(\mathrm{GR})}$ are the momentum density components, and $B^{0\mu}_{(\mathrm{GR})}$ are the components of the energy current density. The Bergmann-Thomson energy-momentum definition satisfies the following local conservation laws

$$\frac{\partial (B^{\mu\nu}_{(\mathrm{GR})})}{\partial x^{\nu}} = 0 \tag{6}$$

in any coordinate system. The energy and momentum components are given by

$$P^{\mu} = \iiint B^{\mu 0}_{(\mathrm{GR})} dx dy dz. \tag{7}$$

Further Gauss's theorem furnishes

$$P^{\mu} = \frac{1}{16\pi} \iint \Pi^{\mu 0\alpha} \kappa_{\alpha} dS. \tag{8}$$

where κ_{α} stands for the 3-components of unit vector over an infinitesimal surface element dS. The quantities P^i for $i = 1, 2, 3$ are the momentum components, while P^0 is the energy.

2.2 Einstein Energy-Momentum Prescription in GR

The energy-momentum complex as defined by Einstein [18] is given by

$$E^{\nu\mu}_{(\mathrm{GR})} = \frac{1}{16\pi} H^{\nu\mu\alpha}_{,\alpha} \tag{9}$$

where

$$H^{\nu\alpha}_{\mu} = \frac{g_{\mu\beta}}{\sqrt{-g}} [-g(g^{\nu\beta} g^{\alpha\xi} - g^{\alpha\beta} g^{\nu\xi})]_{,\xi}, \tag{10}$$

$E^{00}_{(\mathrm{GR})}$ is the energy density, $E^{\alpha 0}_{(\mathrm{GR})}$ are the momentum density components, and $E^{0\alpha}_{(\mathrm{GR})}$ are the components of energy current density. The Einstein energy and momentum density satisfies the local conservation laws

$$\frac{\partial (E^{\mu\nu}_{(\mathrm{GR})})}{\partial x^{\nu}} = 0 \tag{11}$$

and energy and momentum components are given by

$$P^{\mu} = \iiint E^{\mu 0}_{(\mathrm{GR})} dx dy dz. \tag{12}$$

Further Gauss's theorem furnishes

$$P^{\mu} = \frac{1}{16\pi} \iint H^{0\alpha}_{\mu} \eta_{\alpha} dS. \tag{13}$$

η_{α} stands for the 3-components of unit vector over an infinitesimal surface element dS. The quantities P^i for $i = 1, 2, 3$ are the momentum components, while P^0 is the energy.

2.3 Landau-Lifshitz Energy-Momentum Prescription in GR

Energy-momentum prescription of Landau-Lifshitz [11] is given by

$$L^{\mu\alpha}_{(\mathrm{GR})} = \frac{1}{16\pi} S^{\mu\nu\alpha\beta}_{,\nu\beta} \tag{14}$$

where

$$S^{\mu\nu\alpha\beta} = -g(g^{\mu\alpha} g^{\nu\beta} - g^{\mu\beta} g^{\nu\alpha}), \tag{15}$$

$L^{00}_{(\mathrm{GR})}$ is the energy density, $L^{\mu 0}_{(\mathrm{GR})}$ are the momentum density components, and $L^{0\mu}_{(\mathrm{GR})}$ are the components of energy current density. The Landau-Lifshitz energy-momentum complex satisfies the local conservation laws

$$\frac{\partial (L^{\mu\nu})_{(\mathrm{GR})}}{\partial x^{\nu}} = 0 \tag{16}$$

in any coordinate system. The energy and momentum components are given by

$$P^{\mu} = \iiint L^{\mu 0}_{(\mathrm{GR})} dx dy dz. \tag{17}$$

Further Gauss's theorem furnishes

$$P^{\mu} = \frac{1}{16\pi} \iint S^{\mu\alpha 0\nu}_{,\nu} \eta_{\alpha} dS \tag{18}$$

where η_{α} stands for the 3-components of unit vector over an infinitesimal surface element dS. The quantities P^{i} for $i = 1, 2, 3$ are the momentum components, while P^{0} is the energy.

2.4 Möller Energy-Momentum Prescription in GR

In general relativity, Möller energy-momentum complex [31] is given by

$$M^{\mu\nu}_{(\mathrm{GR})} = \frac{1}{8\pi} \Omega^{\mu\nu\sigma}_{,\sigma} \tag{19}$$

where the antisymmetric superpotential is

$$\Omega^{\nu\sigma}_{\mu} = -\Omega^{\sigma\nu}_{\mu} = \sqrt{-g}\left(\frac{\partial g_{\mu\alpha}}{\partial x^{\beta}} - \frac{\partial g_{\mu\beta}}{\partial x^{\alpha}}\right) g^{\nu\beta} g^{\alpha\sigma}, \tag{20}$$

$M^{00}_{(\mathrm{GR})}$ is the energy density and $M^{\mu 0}_{(\mathrm{GR})}$ are the momentum density components. Also, the energy-momentum complex $M^{\mu\nu}_{(\mathrm{GR})}$ satisfies the local conservation laws:

$$\frac{\partial M^{\mu\nu}_{(\mathrm{GR})}}{\partial x^{\nu}} = 0. \tag{21}$$

Obviously, the energy and momentum of the physical system in four-dimensional background is given by

$$P^{\alpha} = \iiint M^{\alpha 0}_{(\mathrm{GR})} dx^{1} dx^{2} dx^{3} \tag{22}$$

where P_0 and P_{α} denote for the energy and the momentum components, respectively. The energy component is obtained by using the Gauss theorem

$$P^{\alpha} = \frac{1}{8\pi} \iint \Omega^{0\sigma}_{0} \eta_{\sigma} dS \tag{23}$$

where η_{σ} is the outward unit normal vector over an infinitesimal surface element dS.

3 Some Energy-Momentum Prescriptions in the Teleparallel Gravity

3.1 Einstein, Bergmann-Thomson and Landau-Lifshitz Energy-Momentum Prescriptions in TG

The tele-parallel gravity is an alternative approach to gravitation and corresponds to a gauge theory for the translation group based on Weitzenböck geometry [32]. In the theory of the tele-parallel gravity, gravitation is attributed to torsion [33], which plays the role of a force [34], and the curvature tensor vanishes identically. The essential field is acted by a nontrivial tetrad field, which gives rise to the metric as a by-product. The translational gauge potentials appear as the nontrivial item of the tetrad field, so induces on space-time a tele-parallel structure which is directly related to the presence of the gravitational field. The interesting place of tele-parallel gravity is that, due to its gauge structure, it can reveal a more appropriate approach to consider some specific problem. This is the situation, for example, in the energy and momentum problem, which becomes more transparent when considered from the tele-parallel point of view.

The Einstein ($E^{\mu\nu}_{\mathrm{TG}}$), Bergmann-Thomson ($B^{\mu\nu}_{\mathrm{TG}}$) and Landau-Lifshitz's ($L^{\mu\nu}_{\mathrm{TG}}$) energy-momentum complexes in tele-parallel gravity [35] are respectively:

$$hE^{\mu\nu}_{(\mathrm{TG})} = \frac{1}{4\pi}\partial_\lambda(U^{\mu\nu\lambda}), \tag{24}$$

$$hB^{\mu\nu}_{(\mathrm{TG})} = \frac{1}{4\pi}\partial_\lambda(g^{\mu\beta}U_\beta^{\nu\lambda}), \tag{25}$$

$$hL^{\mu\nu}_{(\mathrm{TG})} = \frac{1}{4\pi}\partial_\lambda(hg^{\mu\beta}U_\beta^{\nu\lambda}) \tag{26}$$

where $U_\beta^{\nu\lambda}$ is the Freud's super-potential, which is given by:

$$U_\beta^{\nu\lambda} = hS_\beta^{\nu\lambda} \tag{27}$$

where $h = \det(h^a_\mu)$ and $S^{\mu\nu\lambda}$ is the tensor

$$S^{\mu\nu\lambda} = m_1T^{\mu\nu\lambda} + \frac{m_2}{2}(T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) + \frac{m_3}{2}(g^{\mu\lambda}T_\beta^{\beta\nu} - g^{\nu\mu}T_\beta^{\beta\lambda}) \tag{28}$$

with m_1, m_2 and m_3 the three dimensionless coupling constants of tele-parallel gravity [36, 37]. For the tele-parallel equivalent of general relativity the specific choice of these three constants are:

$$m_1 = \frac{1}{4}, \qquad m_2 = \frac{1}{2}, \qquad m_3 = -1. \tag{29}$$

To calculate this tensor, firstly we must calculate Weitzenböck connection:

$$\Gamma^\alpha_{\mu\nu} = h_a^\alpha\partial_\nu h^a_\mu \tag{30}$$

and torsion of the Weitzenböck connection:

$$T^\mu_{\nu\lambda} = \Gamma^\mu_{\lambda\nu} - \Gamma^\mu_{\nu\lambda}. \tag{31}$$

The energy-momentum complexes of Einstein, Bergmann-Thomson and Landau-Lifshitz in the tele-parallel gravity are given by the following equations, respectively,

$$P^{\mu} = \int_{\Sigma} h E^{\mu 0}_{(\mathrm{TG})} dx dy dz, \tag{32}$$

$$P^{\mu} = \int_{\Sigma} h B^{\mu 0}_{(\mathrm{TG})} dx dy dz, \tag{33}$$

$$P^{\mu} = \int_{\Sigma} h L^{\mu 0}_{(\mathrm{TG})} dx dy dz, \tag{34}$$

P_μ is called the momentum four-vector, P_i give momentum components P_1, P_2, P_3 and P_0 gives the energy and the integration hyper-surface Σ is described by $x^0 = t =$ constant.

3.2 Möller Energy-Momentum Prescription in TG

Möller [31] modified general relativity by constructing a new field theory in the tetrad space. He was able to find a general expression for an energy-momentum complex [14] $M^{\mu\nu}_{(\mathrm{TG})}$ that possesses all the required satisfactory properties and formed its superpotential $\Upsilon^{\nu\alpha}_{\mu}$ using the method of infinitesimal transformations:

$$M^{\mu\nu}_{(\mathrm{TG})} = \Upsilon^{\mu\nu\sigma}_{,\sigma} \tag{35}$$

where the expression for the superpotential of Möller's theory can be written in the form

$$\Upsilon^{\nu\sigma}_{\mu} = \frac{(-g)^{1/2}}{2\kappa} P_{\alpha\beta\rho}{}^{\lambda\nu\sigma} (\Phi^{\beta} g^{\rho\alpha} g_{\mu\lambda} - \Lambda g_{\lambda\mu} \gamma^{\alpha\beta\rho} - (1-2\Lambda) g_{\lambda\mu} \gamma^{\rho\beta\alpha}) \tag{36}$$

where Λ equals to a free dimensionless parameter of teleparallel gravity and κ is the Einstein constant. Also, $P_{\alpha\beta\rho}{}^{\lambda\nu\sigma}$ is

$$P_{\alpha\beta\rho}{}^{\lambda\nu\sigma} = \delta_{\alpha}{}^{\lambda} \xi_{\beta\rho}{}^{\nu\sigma} + \delta_{\beta}{}^{\lambda} \xi_{\rho\alpha}{}^{\nu\sigma} - \delta_{\rho}{}^{\nu} \xi_{\alpha\beta}{}^{\nu\sigma}, \tag{37}$$

$\xi_{\beta\rho}{}^{\nu\sigma}$ is the tensor

$$\xi_{\beta\rho}{}^{\nu\sigma} = \delta_{\beta}{}^{\nu} \delta_{\rho}{}^{\sigma} - \delta_{\rho}{}^{\nu} \delta_{\beta}{}^{\sigma}, \tag{38}$$

Φ_α is the basic vector defined by

$$\Phi_{\alpha} = \gamma^{\beta}{}_{\alpha\beta} \tag{39}$$

and the central role in Möller's theory is played by tensor

$$\gamma_{\mu\nu\sigma} = e_{m\mu} e_{m\nu;\sigma} \tag{40}$$

here semicolon denotes covariant differentiation using the Christoffel symbols. $e_{m\nu}$ is the tetrad field and defined uniquely by $g^{\mu\nu} = e_i{}^{\mu} e_j{}^{\nu} \eta^{ij}$ where η_{ij} is the Minkowski metric. Finally, the energy-momentum in teleparallel gravity are expressed by the surface integral

$$P^{\mu} = \lim_{r\to\infty} \int \Upsilon^{\mu 0\alpha} n_{\alpha} dS \tag{41}$$

with n_α being the unit three-vector normal to surface element dS.

4 Some Energy and Momentum of the Hawking Wormhole in GR and TG

4.1 Bergmann-Thomson, Einstein, Möller and Landau-Lifshitz's Energies and Momentums of the Hawking Wormhole in GR

We can write the Hawking Wormhole metric in spherically symmetric Rindler coordinates [38] as

$$ds^2 = \left(1 - \frac{b^2}{\xi^2}\right)^2 (-g^2\xi^2 \cos^2\theta dt^2 + d\xi^2 + \xi^2 d\Omega^2) \tag{42}$$

the space-time may be obtained from the Hawking wormhole metric written in Cartesian coordinates [39]

$$ds^2 = \left(1 - \frac{b^2}{x^\alpha x_\alpha}\right)^2 \eta_{\mu\nu} dx^\mu dx^\nu \tag{43}$$

by means of the coordinate transformation

$$\begin{aligned} x^1 &= \xi \sin\theta \cos\phi, \qquad x^2 = \xi \sin\theta \sin\phi, \\ x^3 &= \xi \cos\theta \cosh gt, \qquad x^0 = \xi \cos\theta \sinh gt \end{aligned} \tag{44}$$

where x^α are the Cartesian coordinates, b is the wormhole's throat radius (which will be taken of the order of Planck lenght), g is the constant with units of acceleration, $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ and $x^\alpha x_\alpha = \mathbf{x}^2 - (x^0)^2$. From (4) and (5), we have obtained the required components of $\Pi^{\mu\nu\beta}$ for the line element (43) to get Bergmann-Thomson energy and momentum of Hawking wormhole

$$\begin{aligned} &\Pi^{001} = -4\frac{A_x}{A}, \qquad \Pi^{002} = -4\frac{A_y}{A}, \qquad \Pi^{003} = -4\frac{A_z}{A}, \\ &\Pi^{101} = \Pi^{202} = \Pi^{303} = 4\frac{A_t}{A} \end{aligned} \tag{45}$$

where $A = (1 - \frac{b^2}{x^\alpha x_\alpha})$ and x, y, z and t indices represent the derivatives according to x, y, z and t, respectively. Using the above component in (3), we obtain Bergmann-Thomson energy and momentum distributions of Hawking wormhole in GR as follows

$$\begin{aligned} B^{00}_{(\text{GR})} &= \frac{A_x^2 + A_y^2 + A_z^2 - A(A_{xx} + A_{yy} + A_{zz})}{4\pi A^2}, \\ B^{10}_{(\text{GR})} &= \frac{AA_{tx} - A_t A_x}{4\pi A^2}, \\ B^{20}_{(\text{GR})} &= \frac{AA_{ty} - A_t A_y}{4\pi A^2}, \\ B^{30}_{(\text{GR})} &= \frac{AA_{tz} - A_t A_z}{4\pi A^2}. \end{aligned} \tag{46}$$

It is obtained the required components of $H_\mu^{\nu\alpha}$ for the line element (43) to get Einstein energy and momentum of Hawking wormhole

$$\begin{aligned} &H_0^{01} = -4AA_x, \qquad H_0^{02} = -4AA_y, \qquad H_0^{03} = -4AA_z, \\ &H_1^{01} = H_2^{02} = H_3^{03} = -4AA_t. \end{aligned} \tag{47}$$

Substituting (47) in (9), we obtain Einstein energy and momentum distributions of Hawking wormhole in GR as follows

$$
\begin{aligned}
E^{00}_{(GR)} &= -\frac{A_x^2 + A_y^2 + A_z^2 + A(A_{xx} + A_{yy} + A_{zz})}{4\pi A^2},\\
E^{10}_{(GR)} &= \frac{AA_{tx} + A_t A_x}{4\pi A^2},\\
E^{20}_{(GR)} &= \frac{AA_{ty} + A_t A_y}{4\pi A^2},\\
E^{30}_{(GR)} &= \frac{AA_{tz} + A_t A_z}{4\pi A^2}.
\end{aligned}
\tag{48}
$$

From (15), we get the required components of $S^{\mu\nu\alpha\beta}$ for the line element (43) to obtain Landau-Lifshitz energy and momentum of Hawking wormhole

$$
S^{0101} = S^{0303} = S^{0202} = -A^4, \qquad S^{1001} = S^{2002} = S^{3003} = A^4. \tag{49}
$$

Using (49) in (14), we obtain Landau-Lifshitz energy and momentum distributions of Hawking wormhole in GR as follows

$$
\begin{aligned}
L^{00}_{(GR)} &= -\frac{A^2(3A_x^2 + 3A_y^2 + 3A_z^2 + A(A_{xx} + A_{yy} + A_{zz}))}{4\pi},\\
L^{10}_{(GR)} &= \frac{A^2(AA_{tx} + 3A_t A_x)}{4\pi},\\
L^{20}_{(GR)} &= \frac{A^2(AA_{ty} + 3A_t A_y)}{4\pi},\\
L^{30}_{(GR)} &= \frac{A^2(AA_{tz} + 3A_t A_z)}{4\pi}.
\end{aligned}
\tag{50}
$$

From (20), we get the required components of $\Omega_\sigma^{\nu\alpha}$ for the line element (43) to obtain Möller energy and momentum of Hawking wormhole

$$
\begin{aligned}
&\Omega_0^{01} = 2AA_x, \qquad \Omega_0^{02} = 2AA_y, \qquad \Omega_0^{03} = 2AA_z,\\
&\Omega_1^{10} = \Omega_2^{20} = \Omega_3^{30} = -2AA_t.
\end{aligned}
\tag{51}
$$

Using (51) in (19), we obtain Möller energy and momentum distributions of Hawking wormhole in GR as follows

$$
\begin{aligned}
M^{00}_{(GR)} &= \frac{A_x^2 + A_y^2 + A_z^2 + A(A_{xx} + A_{yy} + A_{zz})}{4\pi A^2},\\
M^{10}_{(GR)} &= -\frac{AA_{tx} + A_t A_x}{4\pi A^2},\\
M^{20}_{(GR)} &= -\frac{AA_{ty} + A_t A_y}{4\pi A^2},\\
M^{30}_{(GR)} &= -\frac{AA_{tz} + A_t A_z}{4\pi A^2}.
\end{aligned}
\tag{52}
$$

4.2 Bergmann-Thomson, Einstein, Möller and Landau-Lifshitz's Energy and Momentum of the Hawking Wormhole in TG

If we would like to calculate Einstein, Bergmann-Thomson, Möller and Landau-Lifshitz's energy and momentum distributions of the metric in TG, it is needed to calculate tetrad components of the line element (43). So, we obtain the tetrad components (h_i^{μ}) of line element (43) as

$$h_i^{\mu} = \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & A \end{bmatrix}. \tag{53}$$

From (30) and (53), we have obtained the components of Weitzenböck connections as

$$\begin{aligned}
\Gamma^1_{11} &= \Gamma^2_{21} = \Gamma^3_{31} = \Gamma^0_{01} = \frac{A_x}{A}, \\
\Gamma^1_{12} &= \Gamma^2_{22} = \Gamma^3_{32} = \Gamma^0_{02} = \frac{A_y}{A}, \\
\Gamma^1_{13} &= \Gamma^2_{23} = \Gamma^3_{33} = \Gamma^0_{03} = \frac{A_z}{A}, \\
\Gamma^1_{10} &= \Gamma^2_{20} = \Gamma^3_{30} = \Gamma^0_{00} = \frac{A_t}{A}.
\end{aligned} \tag{54}$$

Using (54) in (31), we have calculated the torsion of the Weitzenböck connections as follows

$$\begin{aligned}
T^2_{12} &= T^3_{13} = T^0_{10} = -T^2_{21} = -T^3_{31} = -T^0_{01} = \frac{A_x}{A}, \\
T^1_{21} &= T^3_{23} = T^0_{20} = -T^1_{12} = -T^3_{32} = -T^0_{02} = \frac{A_y}{A}, \\
T^1_{12} &= T^2_{32} = T^0_{30} = -T^1_{21} = -T^2_{23} = -T^0_{03} = \frac{A_z}{A}, \\
T^1_{01} &= T^2_{02} = T^3_{03} = -T^1_{10} = -T^2_{20} = -T^3_{30} = \frac{A_t}{A}.
\end{aligned} \tag{55}$$

Using (27–31) and (53–55), the required components of Freuds's super-potential are obtained as

$$\begin{aligned}
U_2^{21} &= U_3^{31} = U_0^{01} = -U_2^{12} = -U_3^{13} = -U_0^{10} = -AA_x, \\
U_1^{12} &= U_3^{32} = U_0^{02} = -U_1^{21} = -U_3^{23} = -U_0^{20} = -AA_y, \\
U_1^{13} &= U_2^{23} = U_0^{03} = -U_1^{31} = -U_0^{30} = -U_2^{32} = -AA_z, \\
U_1^{01} &= U_2^{02} = U_3^{03} = -U_1^{10} = -U_2^{20} = -U_3^{30} = -AA_t.
\end{aligned} \tag{56}$$

Substituting (56) in (24–26), the Einstein, Bergmann-Thomson and Landau-Lifshitz's energy and momentum densities are obtained, as follows in TG, respectively:

$$\begin{aligned} hE^{00}_{(\text{TG})} &= -\frac{A_x^2 + A_y^2 + A_z^2 + A(A_{xx} + A_{yy} + A_{zz})}{4\pi A^2}, \\ hE^{10}_{(\text{TG})} &= \frac{AA_{tx} + A_t A_x}{4\pi A^2}, \\ hE^{20}_{(\text{TG})} &= \frac{AA_{ty} + A_t A_y}{4\pi A^2}, \\ hE^{30}_{(\text{TG})} &= \frac{AA_{tz} + A_t A_z}{4\pi A^2}, \end{aligned} \tag{57}$$

$$\begin{aligned} hB^{00}_{(\text{TG})} &= \frac{A_x^2 + A_y^2 + A_z^2 - A(A_{xx} + A_{yy} + A_{zz})}{4\pi A^2}, \\ hB^{10}_{(\text{TG})} &= \frac{AA_{tx} - A_t A_x}{4\pi A^2}, \\ hB^{20}_{(\text{TG})} &= \frac{AA_{ty} - A_t A_y}{4\pi A^2}, \\ hB^{30}_{(\text{TG})} &= \frac{AA_{tz} - A_t A_z}{4\pi A^2}, \end{aligned} \tag{58}$$

and

$$\begin{aligned} hL^{00}_{(\text{TG})} &= -\frac{A^2(3A_x^2 + 3A_y^2 + 3A_z^2 + A(A_{xx} + A_{yy} + A_{zz}))}{4\pi}, \\ hL^{10}_{(\text{TG})} &= \frac{A^2(AA_{tx} + 3A_t A_x)}{4\pi}, \\ hL^{20}_{(\text{TG})} &= \frac{A^2(AA_{ty} + 3A_t A_y)}{4\pi}, \\ hL^{30}_{(\text{TG})} &= \frac{A^2(AA_{tz} + 3A_t A_z)}{4\pi}. \end{aligned} \tag{59}$$

Furthermore, from (36), we get the required components of $\Upsilon_\sigma^{\nu\alpha}$ for the line element (43) to obtain Möller energy and momentum of Hawking wormhole

$$\Upsilon_0^{01} = -\frac{2AA_x}{\kappa}, \qquad \Upsilon_0^{02} = -\frac{2AA_y}{\kappa}, \qquad \Upsilon_0^{03} = -\frac{2AA_z}{\kappa}. \tag{60}$$

Using (60) in (35), we obtain Möller energy and momentum distributions of Hawking wormhole in TG as follows

$$\begin{aligned} hM^{00}_{(\text{TG})} &= -\frac{A_x^2 + A_y^2 + A_z^2 + A(A_{xx} + A_{yy} + A_{zz})}{4\pi A^2}, \\ hM^{10}_{(\text{TG})} &= -\frac{A_x^2 + A_y^2 + A_z^2 - AA_{tx} - A_t A_x + A(A_{xx} + A_{yy} + A_{zz})}{8\pi A^2}, \\ hM^{20}_{(\text{TG})} &= -\frac{A_x^2 + A_y^2 + A_z^2 - AA_{ty} - A_t A_y + A(A_{xx} + A_{yy} + A_{zz})}{8\pi A^2}, \\ hM^{30}_{(\text{TG})} &= -\frac{A_x^2 + A_y^2 + A_z^2 - AA_{tz} - A_t A_z + A(A_{xx} + A_{yy} + A_{zz})}{8\pi A^2}. \end{aligned} \tag{61}$$

Table 1 Bergmann-Thomson, Einstein, Landau-Lifshitz and Möller energy and momentum distributions for Hawking wormhole in GR and TG

Type	Energy-momentum in GR	Energy-momentum in TG
Bergmann Thomson	$B^{00}_{(\mathrm{GR})} = \frac{A_x^2 + A_y^2 + A_z^2 - A(A_{xx} + A_{yy} + A_{zz})}{4\pi A^2}$	$hB^{00}_{(\mathrm{TG})} = \frac{A_x^2 + A_y^2 + A_z^2 - A(A_{xx} + A_{yy} + A_{zz})}{4\pi A^2}$
	$B^{10}_{(\mathrm{GR})} = \frac{AA_{tx} - A_t A_x}{4\pi A^2}$	$hB^{10}_{(\mathrm{TG})} = \frac{AA_{tx} - A_t A_x}{4\pi A^2}$
	$B^{20}_{(\mathrm{GR})} = \frac{AA_{ty} - A_t A_y}{4\pi A^2}$	$hB^{20}_{(\mathrm{TG})} = \frac{AA_{ty} - A_t A_y}{4\pi A^2}$
	$B^{30}_{(\mathrm{GR})} = \frac{AA_{tz} - A_t A_z}{4\pi A^2}$	$hB^{30}_{(\mathrm{TG})} = \frac{AA_{tz} - A_t A_z}{4\pi A^2}$
Einstein	$E^{00}_{(\mathrm{GR})} = -\frac{A_x^2 + A_y^2 + A_z^2 + A(A_{xx} + A_{yy} + A_{zz})}{4\pi A^2}$	$hE^{00}_{(\mathrm{TG})} = -\frac{A_x^2 + A_y^2 + A_z^2 + A(A_{xx} + A_{yy} + A_{zz})}{4\pi A^2}$
	$E^{10}_{(\mathrm{GR})} = \frac{AA_{tx} + A_t A_x}{4\pi A^2}$	$hE^{10}_{(\mathrm{TG})} = \frac{AA_{tx} + A_t A_x}{4\pi A^2}$
	$E^{20}_{(\mathrm{GR})} = \frac{AA_{ty} + A_t A_y}{4\pi A^2}$	$hE^{20}_{(\mathrm{TG})} = \frac{AA_{ty} + A_t A_y}{4\pi A^2}$
	$E^{30}_{(\mathrm{GR})} = \frac{AA_{tz} + A_t A_z}{4\pi A^2}$	$hE^{30}_{(\mathrm{TG})} = \frac{AA_{tz} + A_t A_z}{4\pi A^2}$

Table 1 (*Continued*)

Type	Energy-momentum in GR	Energy-momentum in TG
Landau Lifshitz	$L^{00}_{(GR)} = \frac{-3A^2(A_x^2 + A_y^2 + A_z^2 + \frac{A}{3}(A_{xx} + A_{yy} + A_{zz}))}{4\pi}$	$hL^{00}_{(TG)} = \frac{-3A^2(A_x^2 + A_y^2 + A_z^2 + \frac{A}{3}(A_{xx} + A_{yy} + A_{zz}))}{4\pi}$
	$L^{10}_{(GR)} = \frac{A^2(AA_{tx} + 3A_tA_x)}{4\pi}$	$hL^{10}_{(TG)} = \frac{A^2(AA_{tx} + 3A_tA_x)}{4\pi}$
	$L^{20}_{(GR)} = \frac{A^2(AA_{ty} + 3A_tA_y)}{4\pi}$	$hL^{20}_{(TG)} = \frac{A^2(AA_{ty} + 3A_tA_y)}{4\pi}$
	$L^{30}_{(GR)} = \frac{A^2(AA_{tz} + 3A_tA_z)}{4\pi}$	$hL^{30}_{(TG)} = \frac{A^2(AA_{tz} + 3A_tA_z)}{4\pi}$
Möller	$M^{00}_{(GR)} = \frac{A_x^2 + A_y^2 + A_z^2 + A(A_{xx} + A_{yy} + A_{zz})}{4\pi A^2}$	$hM^{00}_{(TG)} = -\frac{A_x^2 + A_y^2 + A_z^2 + A(A_{xx} + A_{yy} + A_{zz})}{4\pi A^2}$
	$M^{10}_{(GR)} = -\frac{AA_{tx} + A_tA_x}{4\pi A^2}$	$hM^{10}_{(TG)} = -\frac{A_x^2 + A_y^2 + A_z^2 - AA_{tx} - A_tA_x + A(A_{xx} + A_{yy} + A_{zz})}{8\pi A^2}$
	$M^{20}_{(GR)} = -\frac{AA_{ty} + A_tA_y}{4\pi A^2}$	$hM^{20}_{(TG)} = -\frac{A_x^2 + A_y^2 + A_z^2 - AA_{ty} - A_tA_y + A(A_{xx} + A_{yy} + A_{zz})}{8\pi A^2}$
	$M^{30}_{(GR)} = -\frac{AA_{tz} + A_tA_z}{4\pi A^2}$	$hM^{30}_{(TG)} = -\frac{A_x^2 + A_y^2 + A_z^2 - AA_{tz} - A_tA_z + A(A_{xx} + A_{yy} + A_{zz})}{8\pi A^2}$

5 Summary and Discussion

The definition of energy-momentum localization in both the general theory of relativity and teleparallel gravity has been very exciting and interesting and a controversial problem. Energy-momentum complexes provide the same acceptable energy-momentum distribution for some systems. However, for some systems [40, 41] these prescriptions disagree.

The object of present paper is to show that it is possible to solve the problem of localization of energy and momentum in general relativity and teleparallel gravity by using the energy and momentum complexes.

In this study, we have used different energy-momentum complexes, Bergmann-Thomson, Einstein, Landau-Lifshitz and Möller to calculate energy-momentum of Hawking wormhole. From Table 1, it can be seen that the energy-momentum densities are finite and well defined. We have found that these definitions do not provide the same energy and momentum densities in GR. We were hoping that the theory of teleparallel gravity would solve this problem. Unfortunately, these definitions do not provide also the same results for the energy and momentum densities of Hawking wormhole. We have found the same energy distributions which are different from zero for Hawking wormhole in GR and TG by using Bergmann-Thomson, Einstein, Landau-Lifshitz and Möller definitions. Furthermore, different energy-momentum complexes do not provide the same energy and momentum densities neither in general relativity nor in teleparallel gravity. We note that both general relativity and teleparallel gravity are equivalent theories, that is, the Einstein, Bergmann-Thomson and Landau-Lifshitz energy and momentum densities of Hawking wormhole are the same and different from zero in both theories. From above results, we have concluded that energy is localized to the region where the energy-momentum tensor is non-vanishing. Also, our results support Lessner's [19] view that Möller energy-momentum complex is the powerful concept to calculate energy distribution in a given space-times.

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